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A numerical study of active flow control of a separated converging-diverging channel flow is presented. Several geometrical and operating parameters of the continuous and pulsed jets are investigated using large-eddy simulations (LES), which are highly resolved in the actuation region. The LES without control is validated with a direct numerical simulation of the same flow. Configurations with both counter-rotating and co-rotating jets are tested. The main statistical results are in general agreement with the literature. Thanks to the unsteady highly resolved simulation, both temporal and spatial organizations of the generated vortices are investigated. An optimal frequency associated with the convective time as well as an optimal duty cycle are obtained, leading to a control efficiency comparable to the cases with continuous jets and the same operating velocity and, consequently, to a significant saving in mass flow rate.

Keywords: turbulence; active control; pulsed jets

Nomenclature

$C_f$ Friction coefficient
$\Delta C_f$ Coefficient of robustness
$C_p$ Pressure coefficient
$E$ Coefficient of efficiency
$F$ Pulsating frequency of the jets
$Fl$ Mass flux from the actuator in percentage of the mass flux of the boundary layer
$G(t)$ Time evolution of the jet amplitude
$h$ Half the channel height (reference length)
$L$ Distance between the two jets of the same pair
$N_j$ Number of jets
$P$ Pressure at the wall
$P_o$ Pressure at the inlet
$Re_{\tau}$ Reynolds number based on the wall friction velocity $u_{\tau}$
$Re_h$ Reynolds number based on half the channel height $h$
$u_{\tau}$ Wall friction velocity
$u_{\tau o}$ Wall friction velocity at the summit of the bump ($x = 0$)

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1. Introduction

Delaying or preventing turbulent boundary layer separation in aeronautic applications (during take-off/landing or manoeuvre of an aircraft, for example) leads to enhance the lift to drag ratio and thus to fuel saving and diminution of the emitted pollution. For these reasons, flow separation control has been the subject of strong attention in the last 50 years. In particular, many types of actuators were explored in the last few decades, over many
different flow configurations (such as flat plates, ramps, bumps, ducts, airfoils, and wind turbines) [1].

Among the variety of actuators used for that purpose, the vortex generators (VGs) were found efficient to reduce and even sometimes suppress the separated region [2]. These devices generate a streamwise vortex structure which can entrain high-momentum fluid toward the wall, hence energizing the turbulent boundary layer, increasing quantities such as wall shear stress, turbulence intensities, and momentum transfer, and delaying the separation. The optimal device would produce streamwise vortices strong enough to overcome the separation without unnecessarily persisting within the boundary layer once the flow control objective is reached. It can be either passive or active. The resulting streamwise vortices have been the subject of several studies, which helped to characterize the optimal actuator parameters and the resulting mean and turbulent organization (see for instance [2–14]).

Passive devices were rapidly replaced by active devices which can be turned off when not necessary in order to avoid additional drag (i.e., during cruise flight for instance). Also, active devices allow to implement different control strategies to adapt the actuation to the baseline flow situations. Many active device types were thus developed and used in continuous and pulsed operating modes.

For continuous jets, the origin of the streamwise vortical structures in the continuous jet case comes from the high shear from the jet edges, which induces a roll-up of the boundary layer flow [15, 16]. When the jet is inclined to the wall and/or to the incoming baseline flow ($\alpha$ skew and/or $\beta$ pitch angle, respectively), two counter-rotating vortices are initially created just downstream of the device which evolves rapidly into an enhanced single coherent vortex of one sign accompanied by a much smaller and weaker region of circulation of the opposite sign near the wall [14]. When the jets depart from the normal to the wall, the skin friction increases, which makes them more robust for flow control separation [7]. The optimal pitch and skew angle is found to be around $\alpha = \beta = 45^\circ$ [5, 7]. Several studies were performed with a single jet arrangement in a zero pressure gradient configuration to analyze the streamwise evolution of the vortices (among others, [14, 17]). However, this does not allow us to take into account the interaction between the streamwise vortices, which is found of major importance [7]. Therefore, many studies were performed with a spanwise distribution of the jets with a co-rotating (CoR) or a counter-rotating (CtR) arrangement (see Figure 6) [4, 7–9, 16, 18]. In a CoR arrangement (i.e. skew angle similar for all jets), the resulting mean streamwise vortices have the same sign of vorticity while, in the CtR arrangement (i.e. the opposite skew angle for each jet of a pair), jets produce pairs of counter-rotating streamwise vortices. A distance between devices of $\lambda/\Phi = 6$ for CoR jets was found to significantly increase the skin friction for a bump configuration [7] (where $\Phi$ is the diameter of round jets). Also, CtR arrangements were found more efficient than CoR [7]. The CtR arrangement generates stable counter-rotating streamwise vortices which stay coherent and individualized far downstream, contrary to co-rotating jets which merged rapidly to form a large vortical region [4]. Finally, increasing the velocity ratio (VR, the ratio of the jet outlet velocity to the external velocity) increases the shear from the jet edges, which has been demonstrated to produce stronger streamwise vortices, i.e. a greater circulation [14–16]. However, too high exit velocity of the jets can blow the produced vortices out of the boundary layer and reduce the control efficiency [14].

In the pulsed mode, the operating parameters of the resulting jet excitation (such as frequency bandwidth, amplitude or VR, duty cycle or phase) are usually used as inputs for different control strategies. Depending on the main flow configuration (i.e. attached or separated turbulent boundary layer over a ramp, a bump, etc.) and the actuator-jet strength,
the pulsating frequency may interact favorably with the baseline flow. These strategies can be classified into three categories: the robust control (high excitation amplitude and adapted frequencies), the control of the baseline instabilities or turbulent structures in turbulent flows (lower excitation amplitudes and adapted frequencies), and the control of turbulent structures at a Kolmogorov scale (very low excitation and high frequencies). The main goal of all these strategies is, for the same control efficiency, to reduce the energy expense of the control which is of major importance for realistic applications. Using pulsed jets with round orifices and a high excitation amplitude \( VR > 1 \), Tilmann et al. [14] and Kostas et al. [8] have reported that the pulsating frequency and the duty cycle (DC) can be tuned so that successive streamwise vortex segments can merge downstream, producing quasi-continuous streamwise vortices with a less mass flow rate than in the steady case. Kostas et al. [8] and Ortmanns and Kaehler [19] and Johari and Rixon [30] have recently reported that the opening and closing of the valve create an enhanced mixing mechanism which significantly increases the skin friction gain and which is not observed for a continuous jet configuration. Kostas et al. [8] and Braud et al. [20] associate the opening and closing effects to a wave propagation in the tubes behind the valves. Using zero-net mass flux actuators in a slot jet configuration at low jet exit amplitude, preferential excitation frequencies are systematically reported [21–24]. This is associated by the authors with a resonance between the produced structures and the structures from the Kelvin–Helmholtz instability of the separated baseline flow. However, turbulent separated flows are sensitive to a wide range of frequencies which are not yet fully understood. In studies using active vortex generator actuators, only few authors have reported such a frequency dependence and only for a complex geometry and a relatively high jet exit amplitude \( VR = 2 \) [18].

The present numerical study (large-eddy simulation, LES) aims at extending the knowledge of the influence of active vortex generator actuators for separation control purpose. To the authors’ knowledge, this is the first LES study on active vortex generators. Most of the numerical studies were performed for zero-net mass flux actuators [21, 22, 24] and only one study on passive devices was performed recently [25]. Parametric investigations of the influence of geometrical parameters are less numerous for active devices than for passive devices as it is experimentally difficult to modify them. Moreover, it is experimentally difficult to isolate a single excitation parameter [15, 20].

In the present study, the control over a bump similar to [7] is performed at a lower Reynolds number \( Re_h = 12,600 \) compare to \( Re_h \approx 3 \times 10^5 \) for the experimental study of Godard et al. [7]). Firstly, the influence of the geometrical parameters is analyzed for a low authority actuation \( VR < 1 \). Two streamwise positions, expected to be sensitive, are tested. Then the influence of the jet orientation is explored by varying the skew \( \alpha = 45^\circ, 90^\circ, 135^\circ \) and pitch \( \beta = 45^\circ, 70^\circ, 90^\circ \) angles. Secondly, the geometry is fixed and operating parameters are explored. The influence of the jet exit amplitude is analyzed for continuous jets. Using one amplitude of the jet exit, a large range of the different pulsating frequencies is explored together with the DC parameter. Finally, for a fixed configuration, the streamwise evolution of the Reynolds stress components is analyzed and the momentum balance is extracted.

2. Numerical simulations

2.1. Numerical method

The LES and the reference direct numerical simulation (DNS) of converging–diverging channel flow were performed using the MFLOPS3D code developed at LML (Laboratoire
de Mécanique de Lille) for simulation of wall-bounded flows with a curved wall. Instead of writing the Navier–Stokes system in curvilinear coordinates, the wall curvature is obtained by a mathematical mapping of the partial differential operators from physical coordinates to Cartesian ones (see [26] for more details on the numerical algorithm). The spatial discretization uses Fourier modes in the spanwise direction, a Chebyshev collocation is used in the normal direction and fourth-order explicit finite differences are used in the streamwise direction. The main interest of this code is to keep a direct resolution method in addition to high-order spatial discretizations. The three-dimensional incompressible Navier–Stokes equations are made dimensionless using half the channel height $h = \frac{1}{a}$ as reference length. The maximum velocity at the inlet $U_o$ is the reference velocity and hence the Reynolds number is $Re = \frac{h U_o}{\nu} = 12,600$. The Reynolds number is often based also on the wall friction velocity $u_\tau$; $Re_\tau = \frac{h u_\tau}{\nu}$. In the present case, $Re_\tau = 617$ at the inlet.

A detailed analysis of the reference DNS of the uncontrolled case can be found in [27] at the same Reynolds number and in [26] at a lower Reynolds number ($Re_\tau = 395$). The uncontrolled flow slightly separates at the lower wall and is close to separation at the upper wall. However, in the present study, only the lower wall will be investigated in detail. As a large number of parameters will be considered, it is necessary to use LES instead of DNS to limit the computational expenses.

### 2.2. LES validation

A complete LES study of the uncontrolled flow has been performed in [28]. The wall-adapting local eddy-viscosity (WALE) model was found to give results comparable to the dynamic Smagorinsky model, and the two subgrid-scale models were able to reproduce the right separation when the streamwise spatial resolution is high enough. All the LES of the present study are performed with the WALE models using a spatial resolution of $1280(x) \times 129(y) \times 256(z)$, here $x$, $y$, and $z$ being the streamwise, wall-normal, and spanwise coordinates, respectively. As a comparison, the reference DNS case was performed on a $2304 \times 384 \times 576$ mesh for the same size of the simulation domain ($12h \times 2h \times \pi h$). The LES mesh is strongly stretched in the streamwise direction (see Figure 1) in order to have a better resolution of the actuated region, which is almost fully resolved. The LES mesh sizes in wall units are $\Delta x^+ = 11$ and $\Delta z^+ = 7.5$ at the inlet and $\Delta x^+ = 5.7$ and $\Delta z^+ = 10$ at the actuation location ($x = 0$). As for comparison, the spatial resolution of the DNS was $\Delta x^+ = \Delta y^+_\text{max} = 5.1$ and $\Delta z^+ = 3.4$ at the inlet. Because of the use of the Chebyshev collocation, the normal direction is overresolved near the wall as four grid points are located below two wall units (at $x = 0$). The streamwise resolution $\Delta x$ is of the same order or even lower than the spanwise resolution $\Delta z$ because of the lower-order spectral response of the discretization scheme in the streamwise direction.

![Figure 1. 2D grid used for the LES of the uncontrolled and controlled flow (1/4 of grid points are plotted in each direction).](image-url)
In order to obtain converged statistics, all the LES (with and without control) are initialized with a DNS velocity and pressure field, and are integrated over 10 convective times (based on the inlet maximum velocity ($U_o = 1$) and half the channel height ($h = 1$)) for most of the cases. Only the last six convective times are retained to compute statistics. However, few cases were integrated twice as long in order to converge some specific statistics such as the phase-averaged friction coefficients and the visualization of coherent structures. The inlet flow is interpolated from the same inlet data (recorded from a precursor channel DNS without bump) that was used for the DNS.

In order to validate the LES, the time- and spanwise-averaged friction coefficient ($C_f$) and pressure coefficient ($C_p$) are compared with the DNS (Figure 2). The two coefficients are defined by

$$C_p = \frac{P - P_o}{\frac{1}{2} \rho U_o^2}, \quad C_f = \frac{\tau_w}{\frac{1}{2} \rho U_o^2},$$

where $\rho = 1$, $\tau_w$ is the wall shear stress averaged in time and in the spanwise direction, $P$ is the wall pressure, and $P_o$ and $U_o$ are the reference pressure and reference streamwise velocity, respectively, at the inlet ($y = h$). A comparison with the DNS is rather accurate except within the separation region, for negative values of the skin friction, where the intensity of the wall shear stress is slightly overestimated by the LES. However, the position of the separation and the reattachment point are predicted accurately. The comparison of the mean streamwise velocity and turbulent kinetic energy is shown in Figure 3 in the diverging part of the flow. The agreement between DNS and LES is very good, even in the separation region.

In the following, $C_f$ and $C_p$ coefficients from the LES with actuation will be systematically compared with the same quantities from the LES without actuation to highlight the benefit of the control. All the distributions of $C_f$ and $C_p$ are averaged in time and in the spanwise direction except for the two-dimensional statistics of Figure 17.
Figure 3. Comparison of (a) the mean streamwise velocity and (b) kinetic turbulent energy between the DNS (black) and the LES (red) of the uncontrolled converging–diverging flow.

2.3. Baseline flow

The baseline flow is a converging–diverging channel flow, which can be described from the friction and pressure coefficient distributions. The curved lower wall creates a converging part. This, until the summit at $x = 0$, induces an acceleration of the flow and thus a favorable pressure gradient (see Figure 2). In the converging part, the “boundary layer thickness” decreases until the point of maximum skin friction coefficient at $x = -0.75$. From this position, the boundary layer begins to grow. Also, for $-0.75 < x < 0$ the skin friction decreases while the pressure gradient is still favorable. After the summit, a pressure recovery occurs in two steps owing to a flow separation. The flow separates between $x = 0.5$ and $x = 1.5$ and induces a negative value of the skin friction coefficient. At the minimum of the skin friction coefficient ($x = 1.2$), the pressure gradient is almost zero. Before and after the flow separation, the adverse pressure gradient is of the same order of magnitude. Therefore, two positions are expected to be sensitive to the actuation. The first at $x = -0.664$ is located near the maximum of skin friction where the produced streamwise vortices may take advantage of the growing boundary layer thickness. The second position is at the summit of the bump, where the adverse pressure gradient is growing.
2.4. Actuation method

In all the controlled cases, the actuation is generated by setting a bleeding velocity profile at the wall. The general shape of the velocity profile is given by

\[ V_p(x, z, t) = \begin{cases} \frac{1}{4} G(t) V \left[ \cos \left( 2 \pi \frac{|x - x^p|}{\delta x} \right) + 1 \right] \cos \left( 2 \pi \frac{|z - z^p|}{\delta z} \right) + 1 \right] & \text{if } \begin{cases} |x - x^p| < \frac{\delta x}{2} \\
|z - z^p| < \frac{\delta z}{2} \end{cases} \\
0 & \text{otherwise}, \end{cases} \]

where \( V_p(x, z, t) = |\vec{V}_p(x, z, t)| \) is the modulus of the velocity of the \( p \)th actuator, \( x^p \), \( z^p \), and \( \delta x = \delta z \) are the position of the center and the size of the \( p \)th actuator orifice, respectively. An example of \( V_p(x, z, t = t_e) \) is given in Figure 4 for the orifice size used in the study (\( \delta x = \delta z \approx 0.08 \)). Note that because of the shape of the outlet profile, the effective diameter of the actuator is estimated to be slightly lower than 0.08 by at least 15%. The size of the jet is approximately \( 1.33 \delta_o \), where \( \delta_o \approx 0.06 \) is the boundary layer thickness at the top of the bump. \( G(t) \) is the time evolution of the jet amplitude defined as follows:

\[ G(t) = \left\lfloor \text{mod}(C \times F \times t, C) - \frac{C}{2} \right\rfloor^8, \]  

where \( C \) is a constant to adjust the duty cycle. The duty cycle can be defined as follows:

\[ DC = \frac{T_{on} \times F}{T_{on} + T_{off}}, \]

where \( T_{on} \) is the period of blowing (i.e. when \( G(t) = 1 \)) and \( T_{off} \) the period without actuation (i.e. when \( G(t) = 0 \)). A constant \( C = 4 \) was used
Figure 5. Example of the two types of function $G(t)$ (see Equation (2)) for two frequencies: $F = 2$ (black and blue) and $F = 10$ (red). The black circles indicate the phases used for the phase-average statistics shown in Figure 21.

for $DC = 0.47$, $C = 10$ for $DC = 0.19$, and $C = 6.5$ for $DC = 0.28$. A second type of function $F$ corresponding to a linear ramp was also tested. The two types of functions are given in Figure 5 for two different frequencies ($F = 2$ and $F = 10$).

For numerical stability reasons, smoothed functions are preferred to sharp ones to define the jets in both space and time.

2.5. Control parameters

Because of the computational effort induced by each simulation, only few parameters are investigated in the present study. The investigated range for each parameters is based on previous experimental studies of Godard et al. [7] on the same geometry but at a much larger Reynolds number. Firstly, several geometrical parameters, such as the streamwise location of the jet $x_p^*$, the jet arrangement (CoR jets or CtR jets), the spanwise distance between jets $\lambda$ and $L$, and the jet orientation ($\alpha$ and $\beta$), are briefly investigated (see Figure 6). Then, based on the best geometrical parameters, the influence of the operating parameters ($V$, $F$, and $DC$) is analyzed. All the tested configurations are listed in Table 1. The actuation is distributed in the full span.

2.6. Robustness and efficiency of the control

The main objective of this study is to investigate the effect of several geometrical and operating parameters. Because of the large number of investigated parameters, it would be unrealistic to find the optimal configuration among the large number of possibilities. Defining an optimum configuration would also require a universal definition of the best
Table 1. List of all controlled cases with their corresponding parameters. The geometrical parameters \((x_a, \lambda, L, \alpha, \beta)\) are defined in Figure 6. The other parameters are the integration time \((T_i)\), the number of jets \((N_j)\), the type of arrangement \((Ar)\), the mass flux rate of the actuation in percentage of the mass flux of the boundary layer at \(x = 0\) \((Fl)\), the frequency \((F)\) used in the time function \(G(t)\) defined by Equation (2) (except for the case indicated by an asterisk \((\ast)\), which uses the ramp function shown in Figure 5), the increase of the skin friction \(\Delta C_f\) defined by Equation (3) and the estimated efficiency \((E)\) defined by Equation (4).

<table>
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<th>(Ar)</th>
<th>(\lambda/L)</th>
<th>(V)</th>
<th>(Fl)</th>
<th>(\alpha)</th>
<th>(\beta)</th>
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<th>(DC)</th>
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<td>90</td>
<td>45</td>
<td>2</td>
<td>0.28</td>
<td>58.3</td>
<td></td>
</tr>
<tr>
<td>P9</td>
<td>10</td>
<td>+0.000</td>
<td>8</td>
<td>CtR</td>
<td>4</td>
<td>1.0</td>
<td>1.7</td>
<td>90</td>
<td>45</td>
<td>3</td>
<td>0.28</td>
<td>65.5</td>
<td></td>
</tr>
<tr>
<td>P10</td>
<td>10</td>
<td>+0.000</td>
<td>8</td>
<td>CtR</td>
<td>4</td>
<td>1.0</td>
<td>1.1</td>
<td>90</td>
<td>45</td>
<td>10/3</td>
<td>0.19</td>
<td>40.4</td>
<td></td>
</tr>
</tbody>
</table>

Figure 6. Geometrical parameters of a pair of counter-rotating jets. Here \(\alpha\) is the skew angle, \(\beta\) is the pitch angle, \(L\) is the distance between the two jets of the same pair, \(\lambda\) is the distance between two pairs in the case of CtR arrangement and \(\delta_x\) and \(\delta_z\) are the streamwise and spanwise sizes of the jet’s outlet (see Figure 4 for the definition of the jets). All the distances are non-dimensionalized using \(h = 1\).
benefit of the control. Moreover, the flow separation in the uncontrolled case is rather marginal and many control configurations are able to suppress the spanwise- and time-averaged recirculation region. In order to evaluate the benefit of the control, two criteria will be defined and computed for all the control configurations. (The first criterion $\Delta C_f$ is a measure of the global increase of $C_f$ in the baseline separated flow region from a few jets diameter downstream the location of the actuation ($x = 0.2$) down to the end of the bump ($x = 3.5$). This parameter is defined as

$$\Delta C_f^{\text{case}} = 100 \times \frac{\left(\int_{x=0.2}^{x=3.5} C_f^{\text{case}} dx' - \int_{x=0.2}^{x=3.5} C_f^{\text{no control}} dx'\right)}{\int_{x=0.2}^{x=3.5} C_f^{\text{no control}} dx'},$$

(3)

where $C_f^{\text{case}}$ and $C_f^{\text{no control}}$ are the time- and spanwise-averaged friction coefficients of the considered case and of the reference case (without control), respectively. This parameter is defined to quantify the net increase ratio of $C_f$ in the downstream part of the bump when the control is active. When this parameter increases under the action of actuators, the flow is expected to stand a larger adverse pressure gradient without separation. Therefore, this parameter can be seen as a measure of the robustness of the control. In order to complete this first parameter, an efficiency coefficient ($E$) is defined as

$$E^{\text{case}} = \left(\frac{\Delta C_f^{\text{case}}}{Fl^{\text{case}}}\right) / \left(\frac{\Delta C_f^{C3}}{Fl^{C3}}\right),$$

(4)

where $Fl$ is the mass flux of the actuation in percentage of the mass flux of the boundary layer at $x = 0$ and $\Delta C_f^{\text{case}}$ is the increase of the averaged friction coefficient defined in Equation (3). The efficiency is arbitrarily scaled to unity for case C3 with continuous jets and $V = 1$. This new coefficient is the ratio of a benefit and a cost of the control. The benefit is defined by the skin friction gain necessary to get a positive value of the friction coefficient and therefore a reattachment of the flow for the present configuration. The cost has been simply evaluated as a linear function of the mass flux of the control jets. This is of course one of the possibilities to estimate the cost (it does not take into account the additional power consumption to pulse the jets for instance), but has the advantage to be simple, independent of the technology used for the actuation, and to treat both the continuous-jet and pulsed-jet cases. These two new indicators $\Delta C_f$ and $E$ are listed in Table 1 and their interpretation should help to understand the results of the present study.

3. Results

The aim of the present study is not to find the optimal parameters as too many configurations would be required for a single bump geometry. The targeted objective is rather to analyze the influence of different geometrical (streamwise position of the actuators, jet arrangement and orientation, and so on) and operating parameters (such as amplitude, pulsating frequency, and duty cycle of actuation) in order to obtain a better physical understanding of the control action for the present bump configuration.
3.1. Influence of geometrical parameters

3.1.1. Influence of the streamwise position

Two positions of the actuation were investigated as they are expected to be sensitive. The first position was chosen close to the maximum of the friction velocity \( x \simeq -0.66 \), which corresponds to the position of the minimum boundary layer thickness. The second position is located at the summit of the bump, close to the minimum of the pressure coefficient. The comparison of the friction velocity is shown in Figure 7 for \( V = 1 \) (cases C1 and C3). For the first streamwise position \( x \simeq -0.66 \), the skin friction coefficient is slightly negative around \( x = 1 \), which indicates that a very small separation remains. On the other hand, for the second position \( x = 0 \), the separation is totally suppressed with a significantly higher level of the skin friction. Therefore, the structures generated by jets/free stream interaction are not able to take advantage of the growing boundary layer thickness as expected. This can be explained in the light of the analysis performed in Section 3.3. Indeed, for the first position, the shear stress coefficient, which was found to be the dominant quantity to balance the adverse pressure gradient and then the necessary quantity to maintain the flow attached, presents a peak which does not persist at the position of separation (not shown here). This indicates that the lifetime of the produced vortices from the first position is not long enough to reach the location of the flow separation. Moreover, the shear stress peak has already moved away from the wall. On the other hand, the actuators located at the summit of the bump significantly affect the Reynolds stresses close to the wall down to \( x = 1.0 \). Therefore, between the two streamwise positions, that located at the summit of the bump is the most efficient and is chosen for the investigation of the other parameters.
3.1.2. Influence of the jet arrangement

Jets in CtR (see Figure 6) or CoR arrangements have been found to produce a different organization of the streamwise vortices [4, 7]. For a CoR arrangement, streamwise vortices merge rapidly to form a large vortical region which moves transversely by self-induction. In the CtR arrangement, vortices stay coherent and individualized far downstream. Also, for the same bump configuration, Godard et al. [7] found that the CtR arrangement is more efficient than the CoR.

In the present study, the two configurations are compared with a constant mass flux and the same number of jets (C2 and C3 in Table 1). For the initial spanwise spacing of \( L = \lambda = 5\delta_z \) in both cases, no differences in the friction coefficient were found between the two configurations (not shown here), although the CoR spacing corresponds to the optimal spacing found by [7]. These results indicate that coherent vortices created by each jet from both arrangements do not interact significantly. Indeed, the advantage of the CtR is usually linked to the ability of the neighboring vortices to enhance the circulation while, in the CoR arrangement, they merged rapidly with a decrease of circulation. In the present case, the CoR jets do no interact before \( x = 1 \). The spacing of the CtR arrangement was decreased down to \( L = \lambda/4 = 2.5\delta_z \), which gives exactly the same evolution of skin friction (see Figure 8). This could be attributed to the Reynolds number differences (\( Re_\theta = 27,610 \) corresponding to \( Re_h \simeq 3 \times 10^5 \) in [7]), the different streamwise positions of the jets relative to the minimum of skin friction, and also to the low amplitude of the jet. The last parameter is supposed to be dominant as the jet amplitude of the present study is at least three time lower than that of [7] and induces much smaller streamwise vortices. Indeed, in Section 3.2.1, where the jet amplitude is investigated, higher jet amplitudes are able to modify the size of the streamwise vortices produced and thus the interaction between them (see Figure 17). In the following, only the counter-rotating arrangement is explored.

3.1.3. Influence of the jet orientation

In [15], the origin of the counter-rotating vortex pair from the jet/free-stream interaction when a single jet is perpendicular to the flow is explained by the high shear from the jet edges, which induces a roll-up of the boundary layer flow. Depending on the jet orientation \( \alpha \) and \( \beta \) (see Figure 6), one vortex of the counter-rotating pair may dominate the other, which leads to a single coherent vortex of one sign accompanied by a much smaller and weaker region of circulation of the opposite sign near the wall [14]. When this configuration occurs, the produced vortex is stronger and the control efficiency is improved. Therefore, the orientation of the jet is a parameter able to modify the induced vortices, and thus the intensity of these vortices, which have the ability to be sustained for a longer convective time. In the present study, three pitch angles and three skew angles were tested for a constant jet velocity \( V \). The results are shown in Figures 9 and 10. Keeping the skew angle to 90° (perpendicular to the flow), the lowest pitch angle \( \beta = 45^\circ \) leads to a skin friction which is slightly lower behind the actuators down to \( x = 1.2 \) and almost identical further downstream. The two other pitch angles (\( \beta = 70^\circ \) and \( \beta = 90^\circ \)) give very similar results. This analysis shows the weak influence of the pitch angle. However, the analysis was conducted at a fairly low \( V \) and the conclusion may change with higher values of the jet velocity. Then, keeping the same pitch angle (\( \beta = 45^\circ \)) for the first part of the controlled region \( 0.2 < x < 1.2 \), the most efficient skew angle is found to be \( \alpha = 135^\circ \), which corresponds to a jet blowing upstream while the case with \( \alpha = 45^\circ \) (blowing downstream) is the less efficient.
Figure 8. Friction coefficients $C_f$ defined by Equation (1) for simulations C2 and C3.

Figure 9. Friction coefficients $C_f$ defined by Equation (1) for simulations C4, C7, and C8.
Figure 10. Friction coefficients $C_f$ defined by Equation (1) for simulations C3, C9, and C10.

The difference in the fluidic obstacles generated by the two configurations is clearly highlighted in Figure 11. In comparison with jets with a pitch angle $\beta = 45^\circ$, actuators which are flowing upstream ($\alpha = 135^\circ$) create a stronger fluidic obstacle at $x = 0$ (see Figure 12), which penetrates further in the boundary layer for the same mass flow rate. Consequently, the shear generated by the jet is higher for $\alpha = 135^\circ$ and induces stronger streamwise vortices further downstream (Figure 13). Therefore, blowing upstream can be viewed as an increase of the velocity of the jet with respect to the local mean velocity. As a matter of fact, the effect on the friction coefficient and the pressure coefficient (not shown here) is similar to what has been observed by increasing the velocity (see Section 3.2.1, Figures 14 and 15). In the following, the orientation of the jets is arbitrarily fixed at $\beta = 45^\circ$ and $\alpha = 90^\circ$.

3.2. Influence of the operating parameters
Starting with the analysis of geometrical parameters for continuous jets, the influence of the operating parameters (the jet amplitude, the pulsating frequency, and the duty cycle) is now investigated.

3.2.1. Influence of the jet amplitude
This parameter is usually defined by the velocity ratio $VR = V/U_\infty$, with $V$ the jet amplitude and $U_\infty$ the free-stream velocity. In our case, the indication of the jet amplitude $V$ was preferred to the velocity ratio as the free-stream velocity is not clearly defined. However, the maximum of the streamwise velocity profile at the inlet and the streamwise velocity at the estimated boundary layer thickness $\delta_o$ at $x = 0$ are $U_{\text{max}} = 1.45$ and $U_{\delta_o} = 1.43$, respectively. Consequently, the velocity ratio can be estimated by dividing the parameter $V$ in Table 1 by one of these two possible definitions of $U_\infty$. Choosing, for instance, $U_\infty = U_{\delta_o}$, the range of velocity ratios investigated is approximately from $VR = 1/3$ for $V = 0.5$ to...
$VR = 1.6$ for $V = 2.5$. These values are in the low range of VR when compared to what has already been tested experimentally (see for instance [5, 7, 8, 16]). This range of the velocity ratio corresponds to an actuation mass flux of approximately 3%–9% of the mass flux integrated over the boundary layer thickness and 0.1%–0.3% of the total channel mass flux at the inlet. The comparison of the friction and pressure coefficients is given in Figures 14 and 15, respectively. All the tested configurations are able to remove the negative range of the skin friction coefficient and the zero pressure gradient which are associated with the flow separation. The general evolution of the skin friction coefficient is similar for all the tested values of $V$ except for $V = 2.5$. Behind the jets, at around $x = 0.05$, the skin friction and pressure coefficients are both locally strongly modified and increase almost linearly with $V$. This is directly linked to the jets which are bent and create a local fluidic obstacle over which a region of favorable pressure gradient ($-0.1 < x < 0.1$) followed by a lower adverse pressure gradient is naturally created. At this location, increasing the jet amplitude $V$ increases the shear induced by the jets (see Figure 16), which leads, as expected, to stronger vortices and then to a higher friction at the wall (not shown here). However, the skin friction is not homogeneous (see Figure 17). Increasing the jet amplitude

Figure 11. Mean streamwise velocity in a plane parallel to the flow (aligned with a jet) for cases C9 and C10 (a, b). The red lines indicate a wall distance corresponding to the boundary layer thickness $\delta_o$ at the summit of the lower bump.
from \( V = 1 \) to \( V = 1.5 \) only slightly extends the spanwise gain of \( C_f \) while a large extent of negative friction (i.e. flow separation) remains in between jets from \( x = 0.2 \) to \( x = 1 \). Actually, the minimum of \( C_f \) at \( x = 0.9 \) in Figure 14 is associated with the end of the region of negative skin friction between jets (see Figures 17a, b). For \( V = 2.5 \), the skin friction distribution at the wall shows that stronger vortices are able to reduce the region of
negative $C_f$ between each pair of jets. However, large values of $V$ affect the skin friction after $x = 1$, where regions of negative $C_f$ are recovered (see Figure 17c). The reduction of the skin friction after this location exists for all the velocity ratios, as shown by the skin friction coefficient for $1.2 < x < 4$. Therefore, for a spanwise average gain of the skin friction, a better compromise in the jet distribution might exist. For the same mass flow rate, a configuration with smaller jets and lower $V$, but close enough to each other to avoid
Figure 16. Mean velocity and mean streamwise vorticity (color) in a plane perpendicular to the flow at $x = 0$ for cases C4–C6.

the separation between jets, could be more efficient. However, this configuration was out of reach of our computing capabilities. Moreover, the lifetime of the vortices would be reduced and the control would be efficient along a smaller streamwise extend.

3.2.2. Influence of frequency

Many authors have reported the enhancement of the control efficiency by excitation of the spanwise structures from the separated shear layer which are initiated by a Kelvin–Helmholz instability mechanism [22–24]. In these studies, the excitation was done through a spanwise slot and mainly with zero-net mass flow actuators, which initially produces spanwise structures. However, the present actuation, which produces streamwise vortices, has, a priori, no reason to excite such an instability mechanism. Moreover, in the present configuration, the shear layer induced by the separated flow does not produce coherent spanwise structures. Hence, it is of interest to investigate the effect of the frequency from the present LES studies.

The time evolution of the jet velocity is defined by Equation (2). A large range of frequency was investigated starting from $F = 1$ to $F = 20$, which can be more or less related to the convective and turbulent time scales, respectively. The control efficiency, observed with the friction coefficient distribution along the streamwise direction, is shown
in Figures 18 and 19 for the low-frequency (from $F = 1$ to $F = 5$) and high-frequency ranges (from $F = 5$ to $F = 20$) respectively, using fixed DC and mass flux. The continuous case with the same mass flux (with $V = 0.5$ instead of $V = 1.0$ for pulsed jets) is given as a reference. The first conclusion is that the benefit for the high frequency range is less than for the continuous jets case (see Figure 18 and Table 1). The case with the highest frequency ($F = 20$) presents only slight differences with the continuous case. For $F = 5$ a significant increase of $C_f$ is observed in the vicinity of the jets (from $x = 0.2$ to $x = 0.9$) as compared to continuous jets. However, this gain is not uniform in the streamwise direction. It is significantly reduced from $x = 1$ to $x = 2$ and then almost identical. The efficiency for $F = 5$ is not obvious and only slightly better when compared with the continuous jets. For the lowest frequencies $F = 1$ and $F = 2$, a significant gain is noticeable from $x = 1$ to $x = 2$, which makes these two cases much more efficient than the others. The time spectra of the local normal velocity in the recirculation region for the uncontrolled case indicate that, for the low control frequency range $1 < F < 4$, a natural enhancement of the frequency bandwidth exists for the uncontrolled case (see Figure 20). It is supposed to be associated with an instability mechanism different from a 2D Kelvin–Helmholz instability and recently reported in [29]. In the following, the controlled case for the optimal frequency $F = 2$ is analyzed in more details.
As the $C_f$ distributions represent a time-averaged vision of the control efficiency for the pulsed-jet cases, phase average measurements were performed at four instants of the actuation cycle, indicated by black circles in Figure 5. The first two instants ($t_o, t_o + 0.1$) are taken when the maximal amplitude of the jet velocity is reached, at the beginning and the end of the actuation stage, respectively. The two other instants ($t_o + 0.2, t_o + 0.3$) are chosen in the rest period (without actuation). The phase average of $C_f$ is presented
Figure 20. Time spectra of the local normal velocity $u_n$ in the recirculation region of the DNS of the uncontrolled flow.

In Figure 21. For $t = t_o$, when the jet is activated, the first peak occurs at the actuator location $x = 0$. Then a second peak, which corresponds to the structures from the previous actuating period $t = t_o - 0.5$, appears at $x = 0.25$. This second peak is broader and noisier. At $t = t_o + 0.1$, the peak is stronger and has not moved from the actuation location. This seems to indicate that the generated structures remain aggregated during the blowing action.

Figure 21. Phase averaged of the friction coefficients $C_f$ defined by Equation (1) for simulation P2. The phases corresponding to the statistics are indicated by solid dots in Figure 5.
Figure 22. Friction coefficients $C_f$ defined by Equation (1) for simulations C3, P2, P7 and P8.

For $t = t_o + 0.2$, two peaks are present around $x = 0.1$ and $x = 0.4$. They correspond to the structures formed by two successive actuation cycles which have been convected in the streamwise direction. Hence, the distance from the first peak to the second peak ($d_x \approx 0.3$) and the time interval for one actuation cycle ($d_t \approx 0.5$) give an evaluation of the convection velocity of the structures $d_x/d_t = 0.6$. The convective velocity corresponds to the mean streamwise velocity at $1/4$ of the boundary layer thickness at $x \approx 0.1$. The second peak has a lower energy and is broader in space than the first peak. This can be explained by the fact that the structures have grown and mixed with the turbulent boundary layer [4]. For $t_o + 0.3$ the first peak at $x = 0.9$ corresponds to the peak found for $t = t_o + 0.2$, which has moved downstream with the same convecting velocity $d_x/d_t \approx 0.06/0.1 = 0.6$. Again, a second broader peak is present (near $x = 0.45$) but less intense. From this analysis, it seems that there exists a minimum blowing duration under which the structure cannot be formed. The evaluation of the optimal time duration will be performed in the following through the investigation of the influence of the duty cycle. The improvement of the control for $F = 2$ is then due to an average of the positive and negative effects from the traveling coherent structures produced by pulsed jets while the control for $F = 20$ or $F = 0$ generates vortices which are less regular. The distribution and shape of induced coherent vortices will be investigated further in Section 3.3.3.

### 3.2.3. Influence of the duty cycle

When the jet amplitude is fixed, the effect of the DC can be associated with a mass flux effect. In the previous investigation, the DC was fixed. Hence, departing from the optimal pulsating frequency $F = 2$ and fixed values of the exit velocity ($V = 1$), the DC is investigated with the objective to reduce the mass flux without affecting the control efficiency.

Before studying the effect of the DC, we first checked the influence of the shape of the time evolution $G(t)$. The acceleration associated with the start of the blowing period may be an important parameter for the generation of vortices. In order to study the possible
effect of acceleration, a test case (P3*) was performed with a linear ramp function (time evolution function with the lowest acceleration) and compared, at the equivalent frequency ($F = 2$) and duty cycle ($DC = 0.47$), with the case with a strong acceleration (P3). The two functions $G(t)$ used for cases P3 and P3* are shown in Figure 5. Surprisingly, the two test cases lead to the same spanwise- and time-averaged friction velocity (not shown). This may indicate that the acceleration $dG(t)/dt$ for case P3 is not strong enough to influence the generation of vortices.

The DC was varied using a fixed value of the frequency $F = 2$ and modifying both the period of blowing $T_{on}$ and the period without actuation $T_{off}$. The friction coefficients are compared in Figure 22 for $DC = 0.19$, $DC = 0.28$, and $DC = 0.47$. The results for the three DCs investigated show that the minimum of $C_f$ located near $x = 0.9$ decreases almost linearly with the duty cycle. Therefore, for this frequency, it is not possible to reduce the mass flux (or DC) without decreasing the control robustness. However, the efficiency ($E$) which integrates the saving of mass flux, is increasing for decreasing DC down to the lowest investigated value of DC. Since the increase of efficiency may be a function of the other parameters such as the outlet velocity of the jet, a minimum of efficiency may exist but could be related to a $\Delta C_f$ not sufficient to control the flow with $V = 1$. Then, keeping a constant period of actuation $T_{on}$, the influence of the rest period $T_{off}$ is investigated. In this case both the frequency and the duty cycle are modified. The resulting friction velocity is shown in Figure 24 and the corresponding time function $G(t)$ is shown in Figure 23. This figure indicates that the rest time is increasing from case P4 to case P7, decreasing DC. For all the tested cases, the control is less efficient than the continuous jet case. However, from $DC = 0.47$ (P4) to $DC = 0.28$ (P9), the control benefit is maintained for $0 < x < 0.9$ and even slightly increased for $0.9 < x < 1.8$ while the DC is decreased by 40% (Figure 23). Increasing further the rest time does not allow to maintain the control benefit (see case P7) but the global control efficiency $E$ is slightly improved. The global benefit of case...
Figure 24. Friction coefficients $C_f$ defined by Equation (1) for simulations C3, P4, P7, and P9.

P9 ($DC = 0.28$) over case P4 ($DC = 0.47$) may be due to the fact that there exists a minimum of the rest period $T_{off}$ for the structures to be convected. Finally, departing from a constant $T_{off}$, $T_{on}$ is decreased to investigate the effect of the blowing duration. Again, in this case, both the frequency and the duty cycle are modified. The results are shown in Figure 25 and the corresponding time functions $G(t)$ are shown in Figure 26. Very similar results are found for cases P2 and P9 while the duty cycle is decreased by around 40% (Figure 25). However, decreasing further $T_{on}$ decreases the robustness of the control (from P9 to P10). Therefore, there exists a minimum of the blowing duration $T_{on}$ for the

Figure 25. Friction coefficients $C_f$ defined by Equation (1) for simulations C3, P2, P9, and P10.
structures to be formed while longer actuation periods corresponding to a larger mass flux are useless in terms of robustness and even prejudicial in terms of efficiency. From these last two investigations on the DC effect, we can conclude that the benefit of choosing a lower DC is strongly linked neither to the period of actuation nor to the period of rest, both have an important role in the formation of the structures and need to be optimized to save the mass flux. Note that, in both cases, the investigation end up with the same optimal parameters in terms of both robustness and efficiency (\( F = 3 \) and \( DC = 0.28 \)), which leads to a mass flux gain of 40% as compared to a \( DC = 0.47 \) used for the frequency analysis. The optimal frequency, which is refined in the present investigation to \( F = 3 \), is actually linked to the optimal duration for the structure to be formed and convected downstream before the following one. Of course the optimal DC parameter, which was determined with a constant \( V = 1 \), depends on the jet amplitude and all the other geometrical parameters and baseline flow configuration such as the turbulent boundary layer characteristics or the pressure gradient.

### 3.3. Statistical analysis

#### 3.3.1. Reynolds stress components

The main components of the Reynolds stress tensor are presented in Figure 27 in the local coordinate system \((s, n)\) (\(s\) and \(n\) being the local streamwise and normal components, respectively) and normalized by inner and outer scales of the boundary layer at the summit of the bump \((u_{\tau_0}, \delta_o, U_\delta)\). The results are shown at four streamwise locations, for two cases with continuous jets (C3 and C4), and compared with the uncontrolled case. For the first streamwise position at \( x = -0.05 \), located just upstream of the actuators, the streamwise and spanwise components of the Reynolds stress are the most affected. Also, increasing the jet amplitude mainly increases the level of the Reynolds stress components. Finally, for this position, the peak of the spanwise component is located closer to the wall. The

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Figure 26. Time evolution \( G(t) \) of the pulsed-jet outlet velocity defined by Equation (2) for three duty cycles: \( DC = 0.47 \) (P2, purple), \( DC = 0.28 \) (P9, blue), and \( DC = 0.19 \) (P10, green).
increase of the streamwise component is probably due to the non-zero spanwise velocity component of the jet. The shear stress is not affected yet. At \( x = 0.25 \), corresponding to approximately \( 4\delta_o \) downstream of the actuators, all the Reynolds stress components are largely affected. The dominant term is the streamwise stress \( \langle u_s'u_s' \rangle \). The peak is larger than the peak from the uncontrolled case and located slightly above \( (nu_{to}/\nu \simeq 30, \text{corresponding to } n \simeq \delta_o/2) \). Also, at this location, increasing the jet amplitude slightly moves the peak away from the wall, but does not increase its level. On the contrary, the peak from the other components, which appears closer to the wall than in the uncontrolled case, increases with the jet amplitude. At \( x = 0.50 \), a second peak of \( \langle u_s'u_s' \rangle \) is growing closer to the wall for the two controlled cases when the peak from the uncontrolled case at \( (nu_{to}/\nu \simeq 30 \) does not move significantly. The peak of the shear stress located near \( nu_{to}/\nu \simeq 20 \) has grown significantly and it will be shown in Section 3.3.2 that the larger negative contribution to the Reynolds stress gradient comes from the normal derivative of the shear stress. At the last location (\( x = 1.25 \)), where the flow separates, all the Reynolds stresses are lower than for the uncontrolled case. This seems to indicate that the separation is delayed as highlighted on the distribution of friction coefficient which is clearly dependent on \( V \).

### 3.3.2 Momentum balance

The benefit of the control is linked to its ability to modify the near-wall shape of the boundary layer profile and especially its normal derivative at the wall \( \partial \langle u_s \rangle /\partial n \). In all the present cases with continuous jets, the flow remain attached at any streamwise position.
Figure 28. Balance of the mean streamwise component of the momentum equation for the uncontrolled case and case C3 controlled with continuous jet. The terms are normalized by the friction velocity at the position of the actuation (left and low axis) and with the external velocity \( U_\delta \) at the same reference position (upper and right axis). The streamwise position of each statistics \((x = 0.1, 0.2, 0.5, 0.9)\) has been indicated at the lower right corner of each plot.

Therefore, it is interesting to determine which terms of the mean streamwise velocity equation are affected under this control action. This equation reads in the local coordinate system \((s, n)\):

\[
\left< u_s \right> \frac{\partial \left< u_s \right>}{\partial s} + \left< u_n \right> \frac{\partial \left< u_s \right>}{\partial n} = -\frac{\partial \left< p \right>}{\partial s} - \frac{\partial \left< u'_s u'_s \right>}{\partial s} - \frac{\partial \left< u'_s u'_n \right>}{\partial n} + \nu \frac{\partial^2 \left< u_s \right>}{\partial s^2} + \nu \frac{\partial^2 \left< u_s \right>}{\partial n^2} \tag{5}
\]

The streamwise evolution of terms I–VII of Equation (5) is shown in Figure 28 for the uncontrolled case and the continuous case C3 \((V = 1, \alpha = 90^\circ, \beta = 45^\circ)\). The terms are normalized with the characteristics of the boundary layer at the summit of the bump \((u_{\tau o}, \delta_o, U_\delta_o)\). At the wall, up to \(0.01 \delta_o\) (corresponding to approximately one wall unit), Equation (5) can be reduced to an equilibrium between the pressure gradient (III) and the viscous stress \( (VI + VII) \) similar to the uncontrolled case, as they are dominant for the four streamwise positions. At a wall distance \(0.01 \delta_o < n < 0.2 \delta_o\), the balance evolves with the streamwise position as follows. In the vicinity of the actuation \((x = 0.1)\), the dominant term is the Reynolds stress gradient \((IV \text{ and } V)\), which is balanced mostly by the viscous stresses \((VI \text{ and } VII)\) and partly by the pressure gradient (III). The situation is different at the three other downstream positions, \(x = 0.2, 0.5, \text{ and } 0.9\), where the Reynolds stress gradient is compensated almost completely by a larger pressure gradient. For \(0.2 \delta_o < n < \delta_o\), viscous terms are negligible and the balance of the convection terms is done by the pressure gradient.
and the Reynolds stress gradient. From the above analysis, the negative contribution of the Reynolds stress gradient is found to be an important ingredient of an efficient control. Moreover, from the analysis of each independent term of the Reynolds stress gradient, the shear stress gradient is the most dominant (Figure 29). Hence, actuators which are able to induce shear stresses are the most effective ones to delay separation.

3.3.3. Coherent vortices

Producing a high level of shear stress is not straightforward. Indeed, the spatial and time evolution of the vortices generated by continuous jets are, in the present configuration, more complex than those described in the literature (see for instance [15]). The mean coherent structures created by the counter-rotating jets are highlighted in Figure 30. This figure shows the negative and positive iso-values of the streamwise component of the vorticity computed with the time-averaged velocity for two pairs of counter-rotating jets from case C3 ($F = 0$), P2 ($F = 2$), and P6 ($F = 20$). This figure confirms the complexity of the structures generated by each jet. Concerning the time-averaged vorticity, for $F = 0$, the main vortices visible down to $x = 1$ are the results of several vortices of each sign induced by the shear generated by the jets. The two main elongated vortices from a pair of jets slightly move away from each other, but they are not intense enough to stay alive after $x = 1$, which corresponds to approximately $15\delta_o$. The weakness of the induced vortices is probably due to the low value of the outlet velocity of the jets ($V = 1$). The statistics of the time-averaged vorticity are important as it shows the spatial distribution of negative and positive vorticities. However, the coherent structures generated by the jets are unstable and an instantaneous analysis (right column of Figure 30) shows that the vortices are in fact much smaller and highly intermittent even for actuation with continuous jets. The case at low frequency ($F = 2$) clearly generates intermittent structures as it was expected by looking at the phase-averaged statistics of $C_f$. The distribution of the phase-averaged streamwise vorticity is much more complex compared with the continuous case. The low pulsating frequency leads to a succession of streamwise vortices of the opposite sign. The visualization of the instantaneous vortices using Q-criterion shows a succession of strong vortices and much weaker vortices which are not detected by the chosen iso-value of Q but which are present in the phase-averaged streamwise vorticity. For the highest investigated frequency ($F = 20$), the vorticity computed with the time-averaged velocity
Figure 30. Left: iso-value of positive (blue) and negative (red) streamwise vorticity computed with the time-averaged (C3) or phase-averaged (P2, P6) velocity. Right: iso-value of the Q-criterion at a given time. The spanwise domain is limited to the first two pairs of counter-rotating jets.

exhibits structures similar to the continuous case. The organization of the mean structures is identical, but there intensities are much smaller. The distribution of vortices in the instantaneous picture of Q-criterion is also similar to that generated by continuous jets. The streamwise spacing between vortices is identical and is clearly not related to a frequency of $F = 20$. As expected from the statistics of the friction coefficient, the behavior of high-frequency actuation is very similar to that of continuous actuation with a lower velocity but at an equivalent mass flux.

4. Conclusions
In the present study, an LES investigation of separation control in a converging–diverging channel by means of active pulsed jets was performed. The lower wall of the channel is similar to the bump shape of [7] but at a lower Reynolds number ($Re_\tau = 617$). The influence of the geometrical and operating parameters is explored for a low-authority control (i.e.
VR < 2) with a systematic investigation of the skin friction coefficient gain and the pressure coefficient distribution. Two indicators were defined in order to quantify the benefit of the control. The first is a measure of the net increase ratio of the averaged friction coefficient in the downstream part of the bump. This indicator is called robustness as higher values of this indicator are supposed to be representative of the margin of the controlled case with respect to flow separation. The second indicator measures the efficiency by integrating the cost of the control which has been chosen to be simply a linear function of the mass flux rate used for the control. This choice is not unique but is logical as it can be linearly related to the power consumption.

Two streamwise positions of the actuation were evaluated. The produced vortices were not able to take advantage of the growing boundary layer thickness at the maximum of the skin friction coefficient of the first position and the second position, at the summit of the bump, was more efficient.

In the present study, at a fairly low VR, the pitch angle has no significant effect compared with the skew angle jets. However, this study demonstrates that jets oriented in the opposite direction to the main flow (i.e. \( \alpha = 135^{\circ} \)) are found more efficient than co-flowing jets. This result was already demonstrated in the experiment of Godard et al. [7]. For the same jet velocity \( V \), the relative velocity of the jet to the flow is increased. Therefore, blowing in the upstream direction has a larger impact on the pressure gradient close to the actuators and affects higher regions of the boundary layer.

In the continuous mode, increasing the jet velocity increases the control robustness similar to [9]. However, the skin friction distribution at the wall indicates that this improvement is rather due to a local increase in the control action behind the jets while regions of flow separation remain between the produced streamwise vortices. This is the reason why the robustness coefficient does not linearly increase with the jet velocity. Therefore, in the continuous mode, the efficiency, which is proportional to the jet velocity, is significantly reduced for high-velocity cases (\( E = 1.52 \) for \( V = 0.5 \) to \( E = 0.53 \) for \( V = 2.5 \)).

For a fixed mass flow rate and a low amplitude \( V = 1 \) (i.e. \( VR < 1 \)), an optimal pulsating frequency is found. Only the low range of pulsating frequencies are found to improve the control efficiency, while some high frequencies are found to have a negative effect. The gain for this optimal frequency is associated with the average value of the skin friction from structures traveling in the streamwise direction at a convective velocity, which corresponds to the mean streamwise velocity at 1/4 of the boundary layer thickness near the jet exit. This optimal frequency may be associated with the characteristic of the baseline flow where a generation of vortices induced by an instability has been noticed at the beginning of the adverse pressure gradient region (see [29]). The robustness coefficient of the pulsed case \( P2 (F = 2) \) is almost identical to that of the continuous case \( C3 \) with the same jet velocity and half the mass flux. This corresponds to an efficiency which is more than twice larger for the pulsed case compared with the continuous case with the same jet velocity and 1.5 higher when compared with the continuous case with equivalent mass flux.

For the optimal low-frequency range, low values of the duty cycle are found to reduce the mass flow rate consumption with an efficiency as good as for continuous jets. For \( F = 2 \), the robustness is maintained and the efficiency is improved from \( DC = 0.47 \) to \( DC = 0.28 \). However, for lower DC the robustness decreases but the efficiency is still slightly improving. The benefit of the low duty cycle is not directly related to the time of actuation or the time without actuation but is rather a combination of both. However, a lower limit of the actuation time is expected to maintain a good control efficiency.

Finally, a statistical analysis, for one continuous jet case, highlights that an efficient control is accompanied with an important negative shear stress gradient. However,
producing mainly this quantity is not straightforward. The production of vortices from the active vortex generators embedded in the turbulent boundary layer is not yet fully understood and the present LES simulations highlight the complex instantaneous organization and evolution of the resulting jet/baseline flow interaction, even for continuous jets. Future work will aim at understanding this organization.

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